CAC High School Math Contest – Spring 2022

1. The cost of stainless-steel tubing varies jointly as the length and the diameter of the tubing. If a 5-foot length with diameter 2 inches costs $48.00, how much will a 17-foot length with diameter 4 inches cost?
   A. $324.00
   B. $326.40
   C. $331.70
   D. $331.9
   E. None of the above

2. Two 6-sided dice are rolled. What is the probability that the sum of the two numbers on the dice will be greater than 10?
   A. $\frac{1}{18}$
   B. $\frac{1}{12}$
   C. 3
   D. $\frac{5}{18}$
   E. None of the above

3. In 1999 the stock market took big swings up and down. A survey of 994 adult investors asked how often they tracked their portfolio. The table shows the investor responses. What is the probability that an adult investor tracks his or her portfolio daily?

How frequently?	Response
Daily	227
Weekly	280
Monthly	287
Couple times a year	149
Don't track	51

   A) 22.84%
   B) 24.07%
   C) 28.87%
   D) 28.17%
   E) None of the above

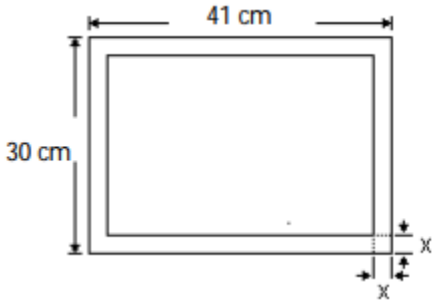
4. A salesman earns $200 weekly plus 5% commission on his weekly sales. If he wants to make at least $740 in a week, how much must his sales be?

A) $\geq \$10{,}800$
B) $\geq \$10{,}780$
C) $\geq \$11{,}600$
D) $\geq \$10{,}850$
E) None of the above

5. A rectangular carpet has a perimeter of 266 inches. The length of the carpet is 95 inches more than the width. Find the dimensions of the carpet.
   A) 114 in. by 19 in.
   B) 76 in. by 95 in.
   C) 114 in. by 133 in.
   D) 123.5 in. by 133 in.
   E) None of the above

6. A manufacturer determines that the profit in dollars for manufacturing n units is $P = 2n^2 - 70n + 50$ (Assume that n is a positive integer) How many units are produced when the profit is \$450?
   A) 50 units
   B) 40 units
   C) 45 units
   D) 5 units
   E) None of the above

7. The outside dimensions of a picture frame are 30 cm and 41 cm. The area of the picture inside the frame is 672 square centimeters. Find the width of the frame.



   A) 18 cm
   B) 9 cm
   C) 4.5 cm
   D) 2.25 cm
   E) None of the above

8. To estimate the number of people in Springfield, population 10,000, who have a swimming pool in their backyard, 250 people were interviewed. Of those polled, 89 had a swimming pool. How many people in the city might one expect to have a swimming pool? (Round your answer to the nearest whole number, if necessary.)

A) 356 people
B) 2 people
C) 28,090 people
D) 3560 people
E) None of the above

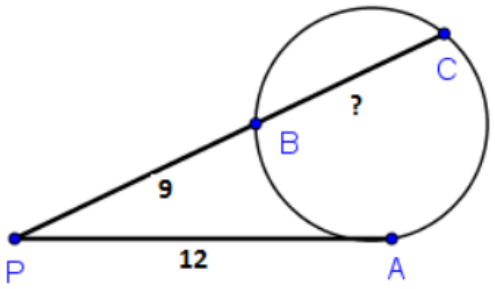
9. Find out how long it takes a $3400 investment to double if it is invested at 8% compounded monthly. Round to the nearest tenth of a year. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.
   A) 8.5 years
   B) 8.7 years
   C) 8.9 years
   D) 9.1 years
   E) None of the above

10. A ladder is resting against a wall. The top of the ladder touches the wall at a height of 15 feet. Find the length of the ladder if the length is 5 feet more than its distance from the wall.
   A) 25 feet
   B) 20 feet
   C) 30 feet
   D) 15 feet
   E) None of the above

11. If the sum of the lengths of all edges of a triangular pyramid is 42 cm, and the edges all have equal length, what is the total surface area of this solid?
   A) $168\ \text{cm}^2$
   B) $49\sqrt{3}\ \text{cm}^2$
   C) $98\ \sqrt{3}\ \text{cm}^2$
   D) $196\sqrt{3}\ \text{cm}^2$
   E) None of the above

12. If $x + y = 5$ and $x^3 + y^3 = 101$ then what is the value of xy?
   A) $\frac{8}{5}$
   B) $\frac{5+\sqrt[3]{101}}{2}$
   C) $\frac{25}{4}$
   D) $5 - \sqrt[3]{101}$
   E) None of the above

13. Consider a circle through points A, B, and C and a point, P, outside that circle. Let segment PA be tangent to the circle and let points P, B, and C be collinear. If $PA = 12$ and $PB = 9$, determine BC.



A) 5
B) 7
C) $3\sqrt{5}$
D) $3\sqrt{7}$
E) None of the above.

14. John's grades in Algebra are: 88 for the homework average, 76 for the quiz average, and 82 for the project average. If homework counts as 10% of the course grade, quizzes count for 40% of the grade, projects count for 20% of the grade, and the final exam counts for the rest, what grade does John need on the final exam to get an 85 average?

A) 85
B) 89
C) 94
D) 98
E) None of the above

15. How many different 5-digit numbers can you form using only the digits 1, 2, 3, and 4, where 4 is used twice and all other mentioned digits are used once?

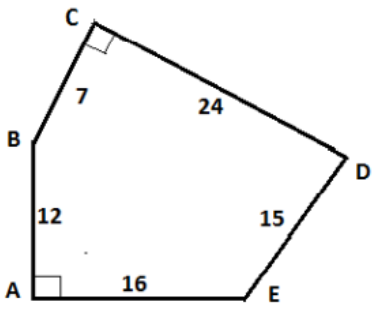
A) 30
B) 55
C) 60
D) 120
E) None of the above

16. Simplify the following expression:

$$\frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}-\sqrt{3}}+\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x}+\sqrt{3}}$$

A) $\frac{2x+6}{x-3}$
B) $\frac{4\sqrt{3x}}{x-3}$
C) $\frac{2x}{x-3}$
D) 2
E) None of the above

17. Consider the pentagon with right angles at vertices A and C, and lengths: $AB = 12$, $BC = 7$, $CD = 24$, $DE = 15$, and $EA = 16$. Determine the area of this pentagon.



A) 74
B) 330
C) 340
D) 360
E) None of the above

18. Find the sum of the two solutions to the equation $9^{x+1} + 3 = 28 \cdot (3^x)$.

A) -3
B) -2
C) -1
D) 1
E) None of the above

19. A kite is a quadrilateral that is symmetric across one of its diagonals. Consider a kite with one right angle and with its two diagonals having lengths $6\sqrt{2}$ cm and $24\sqrt{2}$ cm. What is its perimeter?

A) $4\sqrt{306}$ cm
B) 144 cm
C) $60\sqrt{2}$ cm
D) 72 cm
E) None of the above

20. If $a + b + c + d = 26$, $a - b + c - d = 6$ and $a + b - c - d = 10$ determine $d - 2b + 3c$.

A) 4
B) 14
C) 24
D) 34
E) None of the above

Answers to Level II:

1. B
2. B
3. A
4. A
5. A
6. B
7. C
8. D
9. B
10. A
11. B
12. A
13. B
14. D
15. C
16. A
17. B
18. C
19. D
20. A