

## CAC High School Math Contest 2022 Level III

1. Determine the range of the function  $f(x) = e^{1+x^2}$ .

- a)  $(0, \infty)$
- b)  $(1, \infty)$
- c)  $[1, \infty)$
- d)  $[e, \infty)$
- e)  $(-\infty, \infty)$

2. Simplify the trigonometric expression.

$$2\sec^2 x - 2\sec^2 x \sin^2 x - \sin^2 x - \cos^2 x$$

- a)  $-1$
- b)  $1$
- c)  $0$
- d)  $2\tan^2 x - 1$
- e)  $2\tan^2 x$

3. Let  $b$  and  $c$  be positive integers such that  $\log_b c = 3$  and  $\log_b 4c = 5$ . Find the value of the expression  $b + c$ .

- a) 2
- b) 4
- c) 8
- d) 10
- e) 15

4. A firm sells 100 TV sets per month at \$300 each, but market research indicates that it can sell 1 more set per month for each \$1 reduction of price. At what price (in dollars) will revenue be maximized?

- a) \$100
- b) \$200
- c) \$230
- d) \$250
- e) \$300

5. Let  $a$ ,  $b$ , and  $c$  be the three zeros of  $f(x) = x^3 - 48x + 7$ . What is the value of  $a^3 + b^3 + c^3$ ?

- a)  $-21$
- b)  $-7$
- c)  $21$
- d)  $48$
- e)  $-48$

6. Let  $f(x) = x^5 + x^3 + x + 3$ . Assuming  $f$  is one-to-one, find  $f^{-1}(0)$ .

- a)  $-1$
- b)  $0$
- c)  $3$
- d)  $\frac{1}{3}$
- e) Undefined

7. Let  $f: (0, \infty) \rightarrow \mathbb{R}$  be a function such that

$$3f(x) + 4f\left(\frac{6}{x}\right) = x - 1.$$

Find  $f(12)$ .

- a)  $1$
- b)  $-\frac{2}{7}$
- c)  $-5$
- d)  $\frac{3}{2}$
- e)  $\frac{5}{7}$

8. Find the derivative of  $y = \frac{2}{(3x)^2}$ .

a)  $y' = \frac{1}{3x}$

b)  $y' = -\frac{12}{(3x)^3}$

c)  $y' = -\frac{4}{(3x)^3}$

d)  $y' = -\frac{4}{3x}$

e) None of the above.

9. Suppose that  $3 = k \cdot 2^p$  and that  $15 = k \cdot 4^p$ . What is the value of  $p$ ?

a)  $-\log_2 5$

b)  $\log_5 2$

c)  $\log 5$

d)  $\log_2 5$

e) None of the above

10. Which function value is the smallest?

a)  $\cos \frac{\pi}{18}$

b)  $\sin \frac{\pi}{18}$

c)  $\tan \frac{\pi}{18}$

d)  $\csc \frac{\pi}{18}$

e)  $\sec \frac{\pi}{18}$

11. Determine the  $x$ -value where the graph of  $f(x) = \sin x - \cos x$ ,  $0 \leq x \leq \pi$ , has a horizontal tangent line.

- a) 0
- b)  $\frac{\pi}{2}$
- c)  $\frac{3\pi}{4}$
- d)  $\frac{\pi}{4}$
- e) None of the above.

12. Find the limit:  $\lim_{h \rightarrow 0} \frac{3(1+h)^2 - 3}{h}$

- a) 1
- b)  $\infty$
- c) 3
- d) 6
- e) Does not exist.

13. Three noncollinear points determine a triangle. How many triangles can be formed with 8 noncollinear points?

- a) 336
- b) 24
- c) 56
- d) 6720
- e) None of the above.

14. Find the following:  $\sin\left[\arctan(-1) + \arcsin\left(-\frac{\sqrt{2}}{2}\right)\right]$

- a)  $-\frac{1}{2}$
- b) -1
- c)  $\sqrt{3}$
- d) 1
- e) None of the above.

15. Determine the horizontal asymptote of the function  $f(x) = \frac{200x}{400 + x^2}$

- a)  $y = 0.5$
- b)  $y = 0$
- c)  $y = 200$
- d)  $y = 20$
- e) None of the above.

16. Find the derivative of  $y = \frac{x}{x^2 + 1}$ .

a)  $y' = \frac{1-x^2}{(x^2+1)^2}$

b)  $y' = \frac{1}{2x}$

c)  $y' = \frac{x^2-1}{(x^2+1)^2}$

d)  $y' = \frac{1}{x^2+1}$

e) None of the above.

17. Determine the domain of the function:  $f(x) = \ln(x^2 + 1)$

a)  $(0, \infty)$

b)  $[0, \infty)$

c)  $(-\infty, \infty)$

d)  $[1, \infty)$

e)  $(-\infty, 0) \cup (0, \infty)$

18. If  $f(x) = 6 + 2 \sin(\pi x - \frac{\pi}{2})$ , what is period of  $f(x)$ ?

a)  $2\pi$

b)  $\pi$

c)  $2$

d)  $6$

e)  $8$

19. How many different 4-letter radio-station call letters can be made if the first letter must be K or W, repeats are allowed, but the call letters cannot end in an O?

- a) 33,800
- b) 456,976
- c) 16,900
- d) 35,152
- e) None of the above.

20. Find the second derivative of  $f(x) = \tan^2 x$ .

- a)  $f''(x) = 2 \sec^2 x$
- b)  $f''(x) = 4 \sec^2 x \tan^2 x + 2 \tan^4 x$
- c)  $f''(x) = 4 \sec^3 x \tan x$
- d)  $f''(x) = 4 \sec^2 x \tan^2 x + 2 \sec^4 x$
- e) None of the above.

Answers:

1. d
2. b
3. d
4. b
5. a
6. a
7. c
8. b
9. d
10. b
11. c
12. d
13. c
14. b
15. b
16. a
17. c
18. c
19. a
20. d