MAT 242 Elementary Linear Algebra
Spring 2020 Course Information

Course Description: Introduction to the theories and applications of Linear Algebra. Topics included are systems of linear equations, vectors and matrices, linear transformations, determinants, eigenvectors, eigenvalues, and orthogonality. Prerequisite: MAT221

Measurable Student Learning Outcomes:

1. (Application Level) Solve systems of linear equations using multiple methods, including Gaussian elimination, Cramer's Rule, and matrix inversion. (CSLO #4)

Question 1. Solve the following system using Gaussian elimination.

$$\begin{aligned} x_1 - x_2 - 5x_3 &= -1 \\ -2x_1 + 2x_2 + 11x_3 &= 1 \\ 3x_1 - x_2 + x_3 &= 3 \end{aligned}$$

Question 2. Whice matrix will be used as the inverted coefficient matrix when solving the following system?

$$\begin{aligned} 3x_1 + x_2 &= 4 \\ 5x_1 + 2x_2 &= 7 \end{aligned}$$

(A) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

Question 3. Use Cramer's Rule to solve the following system of equations.

$$\begin{aligned} x + y - z &= 2 \\ 3x - y + z &= 5 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

Question 4. Solve the following system by inverting the coefficient matrix.

$$\begin{aligned} 7x + 2y &= 1 \\ 3x + y &= 5 \end{aligned}$$

2. (Application Level) Compute the transpose, determinant, and inverse of matrices for a given matrix. (CSLO #4)

Question 1. Find all values of λ for which

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

Question 2. Compute the determinant of the following matrix. Simplify completely.

$$\begin{bmatrix} \sin(\alpha)\cos(\beta) & r\cdot\cos(\alpha)\cos(\beta) & -r\cdot\sin(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) & r\cdot\cos(\alpha)\sin(\beta) & r\cdot\sin(\alpha)\cos(\beta) \\ \cos(\alpha) & -r\cdot\sin(\alpha) & 0 \end{bmatrix}$$

3. (Knowledge Level) Define a homogeneous linear system of m equations with n unknowns and identify a sufficient condition for its nontrivial solution. (CSLO #2)

Question 1. Determine whether the following system has no solution, exactly one solution, or infinitely many solutions.

$$\begin{aligned} 2x_1 + 2x_2 &= 2 \\ x_1 + x_2 &= 4 \end{aligned}$$

4. (Application Level) Calculate eigenvalues, eigenvectors and eigenspaces for matrices and linear transformations. (CSLO #4)

Question 1.

Let $A = \begin{bmatrix} -9 & -6 & -22 \\ 1 & 2 & 2 \\ 4 & 2 & 10 \end{bmatrix}$.

(a) Find the characteristic equation of A.

(b) Find the eigenvalues of A.

(c) Find bases for the eigenspaces of A.

5. (Knowledge Level) Define the basic terminology of linear algebra in Euclidean spaces, including linear independence, spanning, basis, rank, nullity, subspace, and linear transformation. (CSLO #2)

Question 1. Are the vectors (1, 2, 4, -3), (1, 1, 0, 1), and (2, 1, 1, 3) in R^4 linearly independent or linearly dependent?

Queation 2. Determine whether the set of all polynomial in the form $a_0 + a_1x + a_2x^2$ where $a_0, a_1, and\ a_2$ are integers is a subspace of P_3. Justify your answer.

6. (Knowledge Level) Find the kernel, rank, range and nullity of a linear transformation. (CSLO #4)

Question 1. Find the rank and nullity of the following matrix.

$$\begin{bmatrix} 1 & 1 & 7 \\ 1 & 2 & 0 \\ 2 & 3 & 10 \end{bmatrix}$$

Question 2. Prove that a square n x n matrix A is invertible if and only rank (A) = n

7. (Application Level) Solve application problems using the properties of linear mappings: image and kernel. (CSLO #4)

Question 1. Let W be the subset of M_{33} of “magic squares”, i.e. 3 x 3 matrices such that for some number c, the sum of any row = the sum of any column = the sum of any diagonal = c. Prove that W is a subspace of M_{33}

8. (Application Level) Use the Gram-Schmidt process to construct orthogonal and orthonormal bases. (CSLO #4)

Question 1. Use the Gram-Schmidt process to transform the basis {(1, 2, -1), (1, 3, 0), (4, 1, 0)} into an orthonormal basis under the Euclidean inner product in R^3.

9. (Application Level) Define subspaces in R-2 and R-3 and inner products; determine the dimension of a subspace and analyze the function that maps two vectors from a vector space to a scalar. (CSLO's #2, #4)

Question 1. Find a basis for the subspace W of M_{22} spanned by

$$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$

Question 2. Determine a basis for the vector space defined by the set of all matrices in the form

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

And determine the dimension of the space.

10. (Synthesis Level) Construct the orthogonal diagonalization of a symmetric matrix. (CSLO #4)

Question 1. Show that the following matrix is orthogonal by showing that the row vectors of the matrix form an orthonormal set in R^n with the Euclidean inner product.

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

Question 2. Find a matrix *P* that orthogonally diagonalizes the following matrix *A*, and determine the value of $D = P^{-1}AP$

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix}$$