MAT241 Analytic Geometry and Calculus III

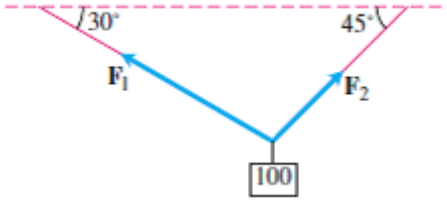
Course Description: Multivariable calculus including vectors, vector-valued functions, partial differentiation, multiple integration, and an introduction to vector fields.

Prerequisite: MAT231 Analytic Geometry and Calculus II

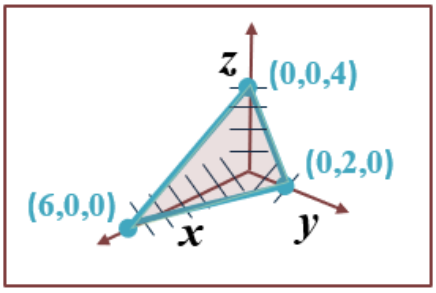
Learning Outcomes and Standards: Upon completion of the course the student will be able to:

1**. (Analysis Level)** Examine the use of vectors in plane and in three-dimensional space.

Example 1. Consider a 100-N weight suspended by two wires as shown. Find the magnitude and components of the force vectors $\mathbf{F}_1$ and $\mathbf{F}_2$.



Example 2. Use the vector cross product to find the equation of the plane shown below.



2. **(Analysis Level)** Describe and compare the motion of an object on a plane or space curve.

Example 1. A projectile is fired is from an initial height of 10 meters with speed of 400 m/sec at an angle of 45°. Determine the maximum height and the range of the projectile.

Example 2. Determine the tangential and normal component of acceleration for the object whose position is given by

$$\mathbf{r}(t) = 4t\,\mathbf{i} + (\cos 2t)\,\mathbf{j} + (\sin 2t)\,\mathbf{k}\,.$$

3. **(Analysis Level)** Analyze the graphs of multivariable functions.

Example 1. Use the second derivative test to find all local maxima, minima,and saddle points of

$$f(x, y) = x^3 - y^3 - 2xy + 3.$$

Example 2. For the function

$$f(x, y) = e^{xy} \sin y\,,$$

find the direction of maximum increase of *f* at the point (1,0). What is the maximum value of the directional derivative of *f* at the point (1,0)?

4. **(Application Level)** Solve real- world applications using multivariable derivative.

Example 1. A box is to have a volume of 125 in^3. Find the dimensions of the box of the smallest surface area.

Example 2. The radius of a right circular cylinder is increasing at a rate of 4 inches per minute, and the height is increasing at a rate of 5 inches per minute. What is the rate of change of the volume when the radius is 8 inches and the height is 10 inches?

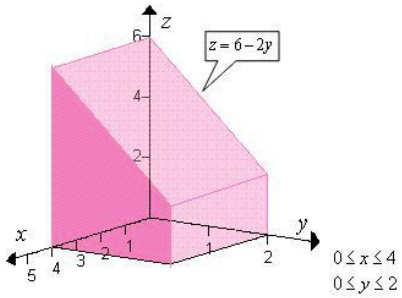
5. **(Evaluation Level)** Select multiple integrals to find characteristic attributes of multidimensional solids.

Example 1. Find the volume of the solid bounded by the paraboloid

$$z = 4 - x^2 - y^2$$

and the *xy*-plane.

Example 2. Find the mass of the solid below with variable density, $\delta(x, y, z) = 2x$.



6. **(Evaluation Level)** Interpret line and surface integrals.

Example 1. Find the work done by the force field **F** on a particle moving along the given path.

$$\mathbf{F}(x, y) = y^2\mathbf{i} + x^2\mathbf{j}$$

$C: x = t, y = t^3$ from (0,0) to (1,1)

Example 2. Use Green's Theorem to find the area of the region enclosed the ellipse

$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (2\sin t)\mathbf{j}.$$

7. **(Synthesis Level)** Incorporate technology to support problem solving processes.

Example 1. Identify and sketch each surface in space. Check by plotting using technology.

a)

$$x^2 + z^2 = 4$$

b)

$$z = 9 - x^2 - y^2$$

c)

$$x^2 - y^2 + z^2 = 1$$

Example 2. Find an equation of the plane that is tangent to the given surface at the given point. Check by plotting the surface and the tangent plane using technology.

$$z = 4x^2 + y^2$$

at the point (1,1,5)