MAT241 Analytic Geometry and Calculus III

Course Description: Multivariable calculus including vectors, vector-valued functions, partial differentiation, multiple integration, and an introduction to vector fields.

Prerequisite: MAT231 Analytic Geometry and Calculus II

Learning Outcomes and Standards: Upon completion of the course the student will be able to:

1. (Analysis Level) Examine the use of vectors in plane and in three-dimensional space.

   Example 1. Consider a 100-N weight suspended by two wires as shown. Find the magnitude and components of the force vectors \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \).

   ![Diagram of Example 1]

   Example 2. Use the vector cross product to find the equation of the plane shown below.

   ![Diagram of Example 2]
2. (Analysis Level) Describe and compare the motion of an object on a plane or space curve.

Example 1. A projectile is fired from an initial height of 10 meters with speed of 400 m/sec at an angle of 45°. Determine the maximum height and the range of the projectile.

Example 2. Determine the tangential and normal component of acceleration for the object whose position is given by
\[ \mathbf{r}(t) = 4t \mathbf{i} + (\cos 2t) \mathbf{j} + (\sin 2t) \mathbf{k}. \]

3. (Analysis Level) Analyze the graphs of multivariable functions.

Example 1. Use the second derivative test to find all local maxima, minima, and saddle points of
\[ f(x, y) = x^3 - y^3 - 2xy + 3. \]

Example 2. For the function
\[ f(x, y) = e^{xy} \sin y, \]
find the direction of maximum increase of \( f \) at the point (1,0). What is the maximum value of the directional derivative of \( f \) at the point (1,0)?

4. (Application Level) Solve real-world applications using multivariable derivative.

Example 1. A box is to have a volume of 125 in\(^3\). Find the dimensions of the box of the smallest surface area.

Example 2. The radius of a right circular cylinder is increasing at a rate of 4 inches per minute, and the height is increasing at a rate of 5 inches per minute. What is the rate of change of the volume when the radius is 8 inches and the height is 10 inches?
5. **(Evaluation Level)** Select multiple integrals to find characteristic attributes of multidimensional solids.

**Example 1.** Find the volume of the solid bounded by the paraboloid

\[ z = 4 - x^2 - y^2 \]

and the \( xy \)-plane.

**Example 2.** Find the mass of the solid below with variable density, \( \delta(x, y, z) = 2x \).

![Diagram]

6. **(Evaluation Level)** Interpret line and surface integrals.

**Example 1.** Find the work done by the force field \( \mathbf{F} \) on a particle moving along the given path.

\[
\mathbf{F}(x, y) = y^2 \mathbf{i} + x^2 \mathbf{j}
\]

\( C : x = t, y = t^3 \) from (0,0) to (1,1)

**Example 2.** Use Green’s Theorem to find the area of the region enclosed the ellipse

\[
\mathbf{r}(t) = (\cos t) \mathbf{i} + (2 \sin t) \mathbf{j}.
\]
7. (Synthesis Level) Incorporate technology to support problem solving processes.

**Example 1.** Identify and sketch each surface in space. Check by plotting using technology.

a) \[ x^2 + z^2 = 4 \]

b) \[ z = 9 - x^2 - y^2 \]

c) \[ x^2 - y^2 + z^2 = 1 \]

**Example 2.** Find an equation of the plane that is tangent to the given surface at the given point. Check by plotting the surface and the tangent plane using technology.

\[ z = 4x^2 + y^2 \]

at the point (1,1,5)