Analytical Geometry and Calculus I

Description: (4 credits) An introduction to analytic geometry, limits, continuity, differential and integral calculus of single variable functions, and related applications.

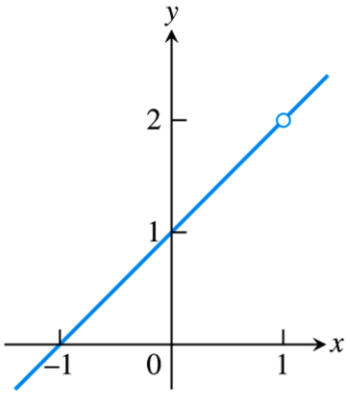
Prerequisites: MAT187; or MAT151 and MAT182; or appropriate test score

Corequisites: None

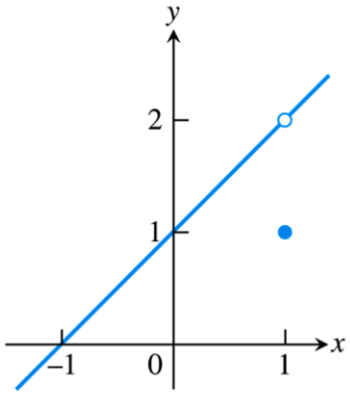
Measurable Student Learning Outcomes

1. (Evaluation Level) Define and evaluate limits algebraically, numerically, or graphically.

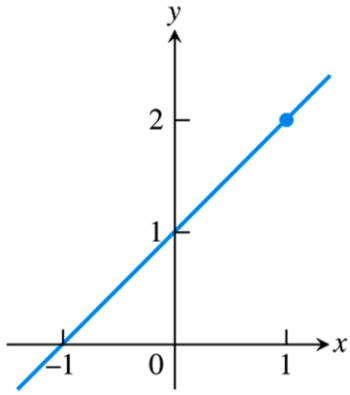
Question 1. Find the limit of the following functions as $x \to 1$



(a) $f(x) = \dfrac{x^2 - 1}{x - 1}$



(b) $g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$

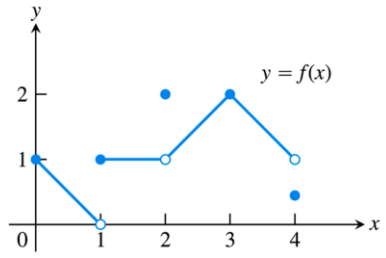


(c) $h(x) = x + 1$

Question 2. Find the limit. $\lim\limits_{x \to 4} \frac{x^2 - 9}{x + 3}$

2. (Knowledge Level) Define various types of discontinuity.

Question 1. At which points do the functions fail to be continuous? At which points, if any, are the discontinuities removable? Not removable? Give reasons for your answers.



Question 2. Show that $f(x) = \frac{x^2+x-6}{x^2-4}, x \neq 2$ has a discontinuity at x = -2.

3. (Evaluation Level) Use multiple methods to differentiate functions.

Question 1. Using the definition, calculate the derivative of the function.
$f(x) = \frac{1}{t^2}; g'(-1), g'(\sqrt{2})$

Question 2. Find the first and second derivatives. $f(x) = 2sin^2x + e^{3x}$

4. (Analysis Level) Use the concepts of calculus to graph a function.

Question 1. Sketch the graph of $f(x) = \frac{x^2+4}{2x}$.

a. Identify the domain of f(x) and any symmetries the curve may have.
b. Find the derivatives y' and y".
c. Find the critical points of f(x), if any, and identify the function's behavior at each one.
d. Find where the curve is increasing and where is decreasing.
e. Find the points of inflection, if any occur, and determine the concavity of the curve.

5. (Application Level) Solve real-world applications using derivative.

Question 1. A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 35 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

6. (Evaluation Level) Approximate the zeros of a function.

Question 1. Use Newton's method to estimate the solutions of the equation $x^2 + x - 1 = 0$. Start with $x_0 = -1$ for the left-hand solution and with $x_0 = 1$ for the solution on the right. Then, in each case, find x_2.

7. (Evaluation Level) Integrate functions.

Question 1. Evaluate the integral. $\int \sqrt{3 - 2x}\, dx$

8. (Evaluation Level) Define and evaluate definite integrals.

Question 1. Use areas to evaluate the integral. $\int_{-2}^{4}(-2x + 4)dx$

9. (Application Level) Solve applications using integral calculus.

Question 1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

10. (Application Level) Incorporate technology to support the problem-solving process.

Question 1. Examine the function $f(x) = \frac{\sqrt{x^2+4}}{3} + \frac{6-x}{5}$ on [0, 6] with a graphing utility and locate all local extrema.