MAT092 Introductory Algebra

Course Description: A review of the basic skills of arithmetic and algebra, with emphasis on developing sound algebraic and problem-solving skills.

Prerequisite: MAT086 Pre-Algebra or MAT087 Foundations I

Learning Outcomes and Standards: Upon completion of the course the student will be able to:

1. (Application Level) Perform the basic operations with polynomials, including add, subtract, multiply, and divide.

> **Example 1.** Subtract the polynomials:
> $$(3x^2 - 3x + 5) - (2x^2 - x - 1)$$

> **Example 2.** Multiply the polynomials:
> $$(x - 2)(x^2 + 3x + 2)$$

> **Example 3.** Divide using long division:
> $$\frac{2x^2 - 6x + 8}{x - 1}$$

2. (Application Level) Apply the laws of exponents, such as squared, cubed, to the power of x based on nested or compound operations, to simplify problems involving the product and/or quotient of expressions with exponents and to express answers without any negative exponents.

Example 1. Simplify using properties of exponents:
$$\left(-3x^3\right)^2$$

Example 2. Rewrite and simplify using <u>positive</u> exponents:
$$\frac{x^{-2}y}{3x^{-1}y^{-2}}$$

3. (Evaluation Level) Evaluate problems involving scientific notation, including converting numbers expressed as scientific notation to standard notation and vice versa.

Example 1. Write in decimal notation:
$$3.4 \times 10^{-4}$$

Example 2. In 2016, outstanding public debt was $\$1.9008 \times 10^{13}$ (which is more than 19 trillion dollars). The population of the United States was approximately 323 million that year. What was each citizen's share of this debt?

4. (Application Level) Apply variation formulas to set up and solve direct and inverse variation problems.

Example 1. Suppose that y varies inversely as x. When $x = 10$, $y = 2$. Find y when $x = 5$.

Example 2. The distance required to stop a car varies directly as the square of its speed. If 250 feet are required to stop a car traveling 60 miles per hour, how many feet are required to stop a car traveling 96 miles per hour?

5. (**Application Level**) Use various factoring techniques to completely factor polynomials, incorporating mathematical concepts such as the greatest common factor and grouping techniques.

      Example 1. Factor by grouping:
$$y^3 + 3y^2 - 2y - 6$$

      Example 2. Factor the trinomial:
$$2x^2 + x - 6$$

      Example 3. Factor completely:
$$9x^5 - x$$

6. (**Application Level**) Solve quadratic equations using factoring and the zero-factor property.

      Example 1. Solve by using the zero factor rule:
$$(2x - 1)(x + 3) = 0$$

      Example 2. Solve by factoring:
$$x^2 = 7x - 6$$

7. (**Application Level**) Perform the basic operations with rational expressions, including determining the domain and range of rational expressions.

      Example 1. Multiply. Write answer in lowest terms.
$$\frac{7x + 7}{x + 4} \cdot \frac{x^2 - x - 20}{7x^2 - 42x - 49}$$

      Example 2. Subtract:
$$\frac{1}{w + 1} - \frac{2}{w}$$

8. **(Application Level)** Construct linear models and graphs given multiple representations of the function and describe trends and predictions for a given data set.

Example. Average annual tuition and fees for in-state students at public four-year colleges are shown in the table for selected years an graphed as ordered pairs of points in the figure, where $x = 0$ represents 2009, $x = 1$ represents 2010, and so on, and y represents the cost in dollars. This graph of ordered pairs of data is called a **scatter diagram**.

Year	Cost (in dollars)
2009	6312
2010	6695
2011	7136
2012	7703
2013	8070

Source: National Center for Education Statistics.
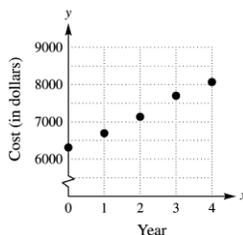
Find an equation that models the data using the data points (1,6695) and (3,7703). Use the equation to predict the cost of tuition and fees at public four-year colleges in 2015.

9. **(Comprehension Level)** Describe the rate of change of a linear function.

Example 1. Find the slope of the line passing through the pair of points:
$$(-1, -2) \text{ and } (1, 6)$$

Example 2. In 2009, Google spent $2800 million on research and development. In 2013, Google spent $8000 million on research and development. Assume a linear relationship and find the average rate of change in the amount of money spent on R&D per year. Graph as a line segment and interpret the result.

10. **(Application Level)** Solve systems of linear equations by using graphing, substitution and elimination methods.

Example 1. Solve the system by using the substitution method:
$$\begin{cases} 2x - y = 5 \\ y = 5x + 1 \end{cases}$$

Example 2. Solve the system by using the elimination method:

$$\begin{cases} 4x + y = 57 \\ 3x - 2y = 18 \end{cases}$$

11. **(Application Level)** Graph linear equations and inequalities in the Cartesian coordinate plane.

Example 1. Graph the linear equation by using the slope and y-intercept.
$$y = -\frac{1}{2}x + 2$$

Example 2. Graph the inequality:
$$3x - 5y \leq 15$$