MAT 187 Pre-Calculus

Credit Hours: 5
Effective Term: Fall 2019
SUN#: 1187
AGEC: Mathematics

Credit Breakdown: 5 Lectures
Times for Credit: 1
Grading Option: A, B, C, D, F
Cross-Listed:

Description: Polynomial, logarithmic, exponential functions, factoring and graphing techniques, angles and their measures, properties and graphs of trigonometric functions and equations, identities, vectors, solutions of triangles, applications, polar coordinates, systems of equations and inequalities, partial fractions, conics, sequences and series. Prerequisites: MAT121 or MAT097 with a grade of B or higher. Prerequisite or corequisite: RDG100.

Prerequisites: MAT121 or MAT097 with a grade of B or higher, and RDG100

Corequisites: RDG100

Recommendations: None

Measurable Student Learning Outcomes & Examples

1. (Application Level) Apply trigonometric concepts to solve right and non-right triangle problems using the law of sines & cosines. (CSLO 2,4)

   Example1:

   A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 6.3 km apart, to be 29° and 52°, as shown in the figure.

   ![Diagram of right triangle with angles 29° and 52° and distance 6.3 km between points A and B]

   NOTE: The picture is NOT drawn to scale.
Find the distance of the plane from point A.
Find the elevation of the plane.

Your answer should be accurate to 2 decimal places.

Example 2:

To find the distance between two radar installations a UFO calculates the distance to installation A to be c=54 km, the distance to installation C to be a=60 km, and the angle between them β=19.1°. Find the distance between the installations.

NOTE: The picture is NOT drawn to scale.

Your answer should be accurate to 2 decimal places.

2. (Application Level) Solve problems involving circles and angles. (CSLO 4)

Example 1:

A record is spinning at the rate of 25 rpm. If a ladybug is sitting 10 cm from the center of the record, find and exact answer for each of the following.

a) What is the angular velocity of the ladybug? (in radians/sec)
b) What is the speed of the ladybug? (in cm/sec)
c) After 21 seconds, how far has the ladybug traveled? (in cm)
d) After 21 seconds, what angle has the ladybug turned through? (in radians)

3. (Application Level) Graph trigonometric functions in rectangular, polar and parametric forms. (CSLO 4)

Example 1:
Given the equation \( y = 2 \sin \left( \frac{7\pi}{6} x + \frac{49\pi}{6} \right) + 6 \)

determine the amplitude, period, horizontal shift and the equation of the midline.

**Example 2:**

A bicycle wheel has radius \( R \). Let \( P \) be a point on the spoke of a wheel at a distance \( d \) from the center of the wheel. The wheel begins to roll to the right along the \( x \)-axis. The curve traced out by \( P \) is given by the following parametric equations:

\[
P = \begin{cases} 
  x(\theta) &= 150 - 7 \sin \theta \\
  y(\theta) &= 15 - 7 \cos (\theta)
\end{cases}
\]

What must we have for \( R \) and \( d \)?

4. (Application Level) Solve trigonometric equations using identities and inverse properties. (CSLO 2,4)

**Example 1:**

Solve \( 2 \cos^2 w - 5 \cos w + 3 = 0 \) for all solutions. Give answers in both radians and degrees.

**Example 2:**

Solve for \( t, 0 \leq t < 2\pi \):

\[18 \sin t \cos t = 8 \sin t\]

5. (Application Level) Use vectors to solve application problems. (CSLO 4)

**Example 1:**

Two children are throwing a ball back-and-forth straight across the back seat of a car. The ball is being thrown 7 mph relative to the car, and the car is travelling 35 mph down the road.

If one child doesn't catch the ball and it flies out the window, in what direction does the ball fly (ignoring wind resistance measured relative to the car's forward direction)?

**Example 2:**

Two ropes hold a crate as shown below. The crate weighs \( F_{gr} = 720 \text{N} \). One rope forms an angle of 35° with the horizontal, and the other one 46° with the horizontal. Compute the tension in each rope. Round your answer to three (or more) decimal places.
6. (Synthesis Level) Develop multiple approaches to solving systems of linear equations. (CSLO 4)

**Example 1:**

Solve the system using elimination, matrices and by using Cramer’s Rule (if possible). Without graphing, state what the graph would look like. Show all work and explain/interpret the results given from each.

\[-2x - 6y = -20\]
\[-3x - 9y = -32\]

**Example 2:**

Using either elimination, matrices or Cramer’s Rule, solve the following. Be sure to show and explain all work.

A movie theater has a seating capacity of 385. The theater charges $5.00 for children, $7.00 for students, and $12.00 of adults. There are half as many adults as there are children. If the total ticket sales was $ 2792, How many children, students, and adults attended?

7. (Application Level) Graph equations of conic sections. (CSLO 4)

**Example 1:**

Identify and graph the given conic section:
\[
\frac{(y - 5)^2}{16} - \frac{(x - 2)^2}{9} = 1
\]

**Example 2:**

Find the focus, directrix, vertex and axis of symmetry for the following parabola and graph: 
\[-8(x + 3) = (y + 1)^2\]

**Example 3:**

Sketch a graph and label all significant information about the graph of
\[
\frac{(x + 1)^2}{4} + \frac{(y + 2)^2}{9} = 1
\]

**Example 1:**

The partial fraction decomposition has the form:
\[
\frac{5x^4 - 34x^3 + 70x^2 - 33x - 19}{(x - 3)^2} = f(x) + \frac{g(x)}{x - 3} + \frac{h(x)}{(x - 3)^2}.
\]

Find \(f(x), g(x),\) and \(h(x)\).

**Example 2:**

Graph the following rational function and clearly identify all of the following features it may contain: vertical asymptotes, horizontal asymptotes, intercepts, holes and end behavior.

\[
f(x) = \frac{x^2}{x^2 - 4}
\]
Example 3:
Solve for the largest value of \( x \) that satisfies:
\[
\log_2(x^2) - \log_2(x + 4) = 6
\]

Example 4:
Solve the given equation for \( x \).
\[
6^{2x-3} = 45
\]

10. (Application Level) Distinguish the difference between arithmetic and geometric sequences and apply appropriate formulas to find specific terms. (CSLO 2,4)

Example 1:
For an arithmetic sequence, \( a_{30} = 87 \). If the common difference is 4, find \( a_1 \) and the sum of the first 34 terms.

Example 2:
Find \( n \) such that the geometric sequence becomes less than 1:
\{600, 360, 216, 129.6, ... \}

11. (Evaluation Level) Evaluate the sum of finite and infinite series. (CSLO 4)

Example 1:
Find the infinite sum, if it exists, for this series:
\[-2 + (0.4) + (-0.08) + \cdots\]

Example 2:
Find the partial sum, \( S_5 \), for the geometric sequence with \( a=5 \) and \( r=3 \).

12. (Evaluation Level) Use technology to model, solve, and justify answers. (CSLO 4)

Example 1:
A ferris wheel is 40 meters in diameter and boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 6 minutes. The function \( h = f(t) \) gives your height in meters above the ground \( t \) minutes after the wheel begins to turn. Find
h(t), graph h(t) and find how high are you off of the ground after 3 minutes being sure to label the point on your graph.

13. (Analysis Level) Apply multiple techniques to solve quadratic equations and other equations reducible to quadratic form. (CSLO 2,4)

   **Example 1:**

   Solve \( e^{2x} + 2e^x = -1 \).

14. (Application Level) Perform the basic operations on matrices. (CSLO 4)

   **Example 1:**

   Add, subtract and multiply the following two matrices, if possible.

   \[
   A = \begin{bmatrix}
   -9 & -8 \\
   2 & -4 \\
   -1 & -6
   \end{bmatrix}, \quad
   B = \begin{bmatrix}
   -3 & -8 \\
   -2 & 7
   \end{bmatrix}
   \]

15. (Evaluation Level) Determine the optimal solution to a function, subject to constraints using a maximum/minimum method. (CSLO 4)

   **Example 1:**

   Caitlyn sells antique plates and antique spoons at the county fair. She wants to sell more than 12 antiques in total. She sells the antique plates for $2.5 and the antique spoons for $1.5. She needs to sell more than $24 worth of antiques in order to earn a profit. Will she make a profit if she sells 6 antique plates and 3 antique spoons? Support your response with a graph and other needed computation.

16. (Application Level) Combine polynomial, rational, and square root functions and find the domain of a composite function. (CSLO 2,4)

   **Example 1:**

   Given functions \( p(x) = \frac{1}{\sqrt{x}} \) and \( m(x) = x^2 - 4 \), state the domains of the following functions using interval notation:

   a) \( \frac{p(x)}{m(x)} \)
   
   b) \( p(m(x)) \)
   
   c) \( m(p(x)) \)