Central Arizona College

2019 Pinal County High School Math Competition

Open

Tom asked his Granny how old she was. Rather than giving him a straight answer, she replied:
“I have 6 children, and there are 4 years between each one and the next. I had my first child (your uncle Peter) when I was 19. Now the youngest one (your Auntie Jane) is 19 herself. That’s all I’m telling you!” How old is Tom’s Granny?

a.) 43

b.) 58

c.) 54

d.) 62

e.) None of the above.

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Expand $\left(\frac{1}{\sqrt{2}}+\frac{\mathrm{i}}{\sqrt{2}}\right)^4$

a.) $1-\mathrm{i}$

b.) 1

c.) -1

d.) $1+\mathrm{i}$

e.) None of the above.

In a lottery game run by a certain state, for every game ticket purchased, a player can pick any 4 numbers from 1 through 42. If the numbers match those drawn by the state, the player wins. If Martin buys 3 tickets, what is his probability of winning?

a.) $\frac{4}{167,895}$

b.) $\frac{1}{111,930}$

c.) $\frac{1}{37,310}$

d.) $\frac{1}{47,970}$

e.) None of the above.

Determine how many of the first 5 terms in the sequence obtainable from the given statement are true.

$$2n < n^2$$

a.) 2

b.) 5

c.) 4

d.) 3

e.) None of the above.

Find numbers a and b, so that f is continuous at every point.

$$f(x) = \begin{cases} x^2, & x < 3 \\ ax + b, & 3 \leq x \leq 5 \\ x + 20, & x > 5 \end{cases}$$

a.) $a = 8, b = 15$

b.) $a = -8, b = -15$

c.) $a = 8, b = -15$

d.) Impossible

e.) None of the above.

Determine the equation of the function that is graphed.

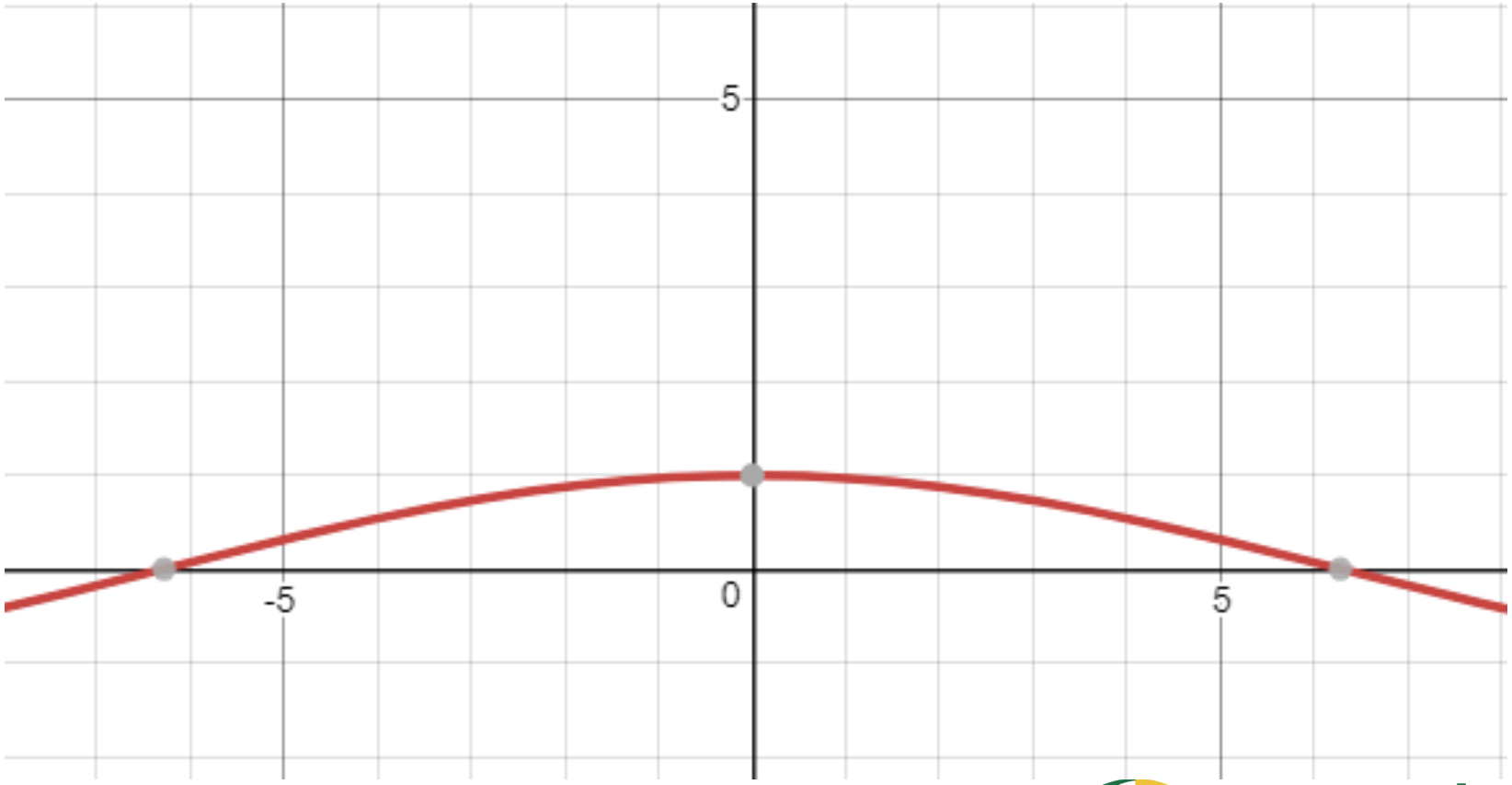
a.) $y = \sin\left(\frac{1}{4}x\right)$

b.) $y = \cos(4x)$

c.) $y = 4 \cdot \sin(x)$

d.) $y = \cos\left(\frac{1}{4}x\right)$

e.) None of the above.



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Use l'Hôpital's rule to find the limit.

$$\lim_{\theta \to 0} \frac{2 - 2 \cdot \cos(\theta)}{\sin(4\theta)}$$

a.) ∞

b.) $\frac{1}{2}$

c.) 0

d.) 1

e.) None of the above.

Suppose that the dollar cost of producing x radios is $c(x) = 400 + 20x - 0.2x^2$. Find the marginal cost when 35 radios are produced.

a.) -$855

b.) $6

c.) $855

d.) $34

e.) None of the above.

Find y''' if $y = 3x \cdot \sin(x)$.

a.) $y''' = 3x \cdot \cos(x) + 9 \cdot \sin(x)$

b.) $y''' = 6 \cdot \cos(x) - 3x \cdot \sin(x)$

c.) $y''' = -3x \cdot \cos(x) + 9 \cdot \sin(x)$

d.) $y''' = -3x \cdot \cos(x) - 9 \cdot \sin(x)$

e.) None of the above.

Find the limit.

$$\lim_{x\to\infty}(\ln(x))^{\frac{4}{x}}$$

a.) 0

b.) e^4

c.) 1

d.) 4

e.) None of the above.

Find $\frac{dy}{dx}$.

$$\ln(y) = e^{y} \cdot \cos(4x)$$

a.) $e^{y} \cdot \cos(4x) - 4e^{y} \cdot \sin(4x)$

b.) $\frac{y \cdot e^{y} \cdot \sin(4x)}{1 - e^{y} \cdot \cos(4x)}$

c.) $-4y \cdot e^{y} \cdot \sin(4x)$

d.) $\frac{-4y \cdot e^{y} \cdot \sin(4x)}{1 - y \cdot e^{y} \cdot \cos(4x)}$

e.) None of the above.

A pulley rotates through 51° in one minute. How many rotations does the pulley make in an hour?

a.) 17.0 rotations

b.) 8.5 rotations

c.) 306.0 rotations

d.) 153.0 rotations

e.) None of the above.

At the given point, find the slope of the curve, the line that is tangent to the curve, or the line that is normal to the curve, as requested.

$$2x^2y - \pi \cdot \cos(y) = 3\pi, \text{ slope at } (1, \pi)$$

a.) π

b.) 0

c.) $\frac{-\pi}{2}$

d.) -2π

e.) None of the above.

Use a finite approximation to estimate the area under the graph of the given function on the stated interval as instructed.

$f(x) = \frac{1}{x}$, between $x = 2$ and $x = 8$ using a lower sum with two rectangles of equal width.

a.) $\frac{39}{40}$

b.) $\frac{21}{40}$

c.) $\frac{39}{10}$

d.) $\frac{21}{10}$

e.) None of the above.

Use Descartes' Rule of Signs to determine the possible number of positive real zeros and the possible number of negative real zeros for the function. $p(x) = 7x^{20} - 3x^{14} - 9x^{11} + 2x^2 - 2x$

a.) 1 or 3 positive; 0 or 2 negative

b.) 1 or 3 positive; 1 or 3 negative

c.) 0 or 2 positive; 0 or 2 negative

d.) 3 positive; 2 negative

e.) None of the above.

The given point is on the graph of $y = f(x)$. Find a point on the graph of $y = g(x)$.

$$g(x) = f(-8x); \quad (3, -8)$$

a.) $(24, 8)$

b.) $(-\frac{3}{8}, -8)$

c.) $(-24, -8)$

d.) $\left(\frac{1}{24}, -3\right)$

e.) None of the above.

At time $t \geq 0$, the velocity of a body moving along the s-axis is $v = t^2 - 8t + 7$. When is the body moving backwards?

a.) $0 \leq t < 1$

b.) $0 \leq t < 7$

c.) $1 < t < 7$

d.) $t > 7$

e.) None of the above.

Company A rents copiers for a monthly charge of $300 plus 10 cents per copy. Company B rents copiers for a monthly charge of $600 plus 5 cents per copy. What is the number of copies above which Company A's charges are the higher of the two?

a.) 6100 copies

b.) 6000 copies

c.) 12,000 copies

d.) 3000 copies

e.) None of the above.

Find the absolute value of the complex number, $2 - 10i$

a.) 12

b.) 104

c.)$2\sqrt{3}$

d.) $2\sqrt{26}$

e.) None of the above.

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Does the graph of the function $y = \tan(x) - x$ have any horizontal tangents in the interval $0 \leq x \leq 2\pi$ If so, where?

a.) Yes, at $x = \frac{\pi}{2}, \frac{3\pi}{2}$

b.) Yes, at $x = 0, x = \pi, x = 2\pi$

c.) Yes, at $x = \pi$

d.) No

e.) None of the above.